CSCI 3210:
Computational Game Theory

www.mtirfan.com/CSCI-3210

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Office Hours:
Wed 3-5:30pm, Fri 10am-12pm

1

Nash equilibrium (?) and its inefficiency

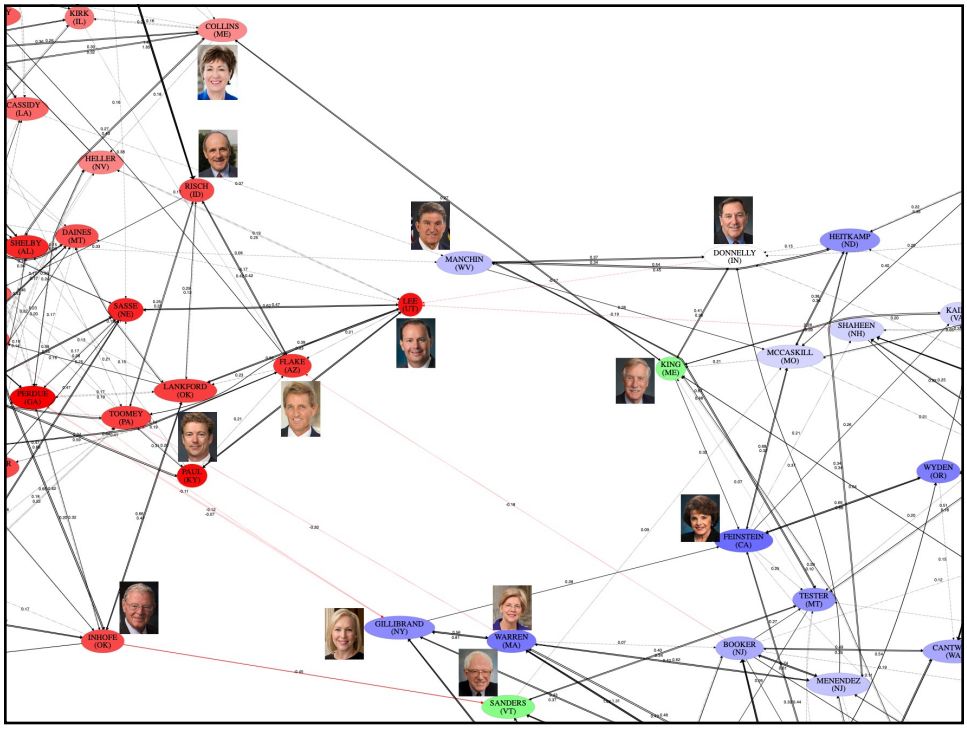
Braess's paradox

Reading: Ch 8 of EK

2


About myself
BOWDOIN
SAILING

4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SANDERS (VT)

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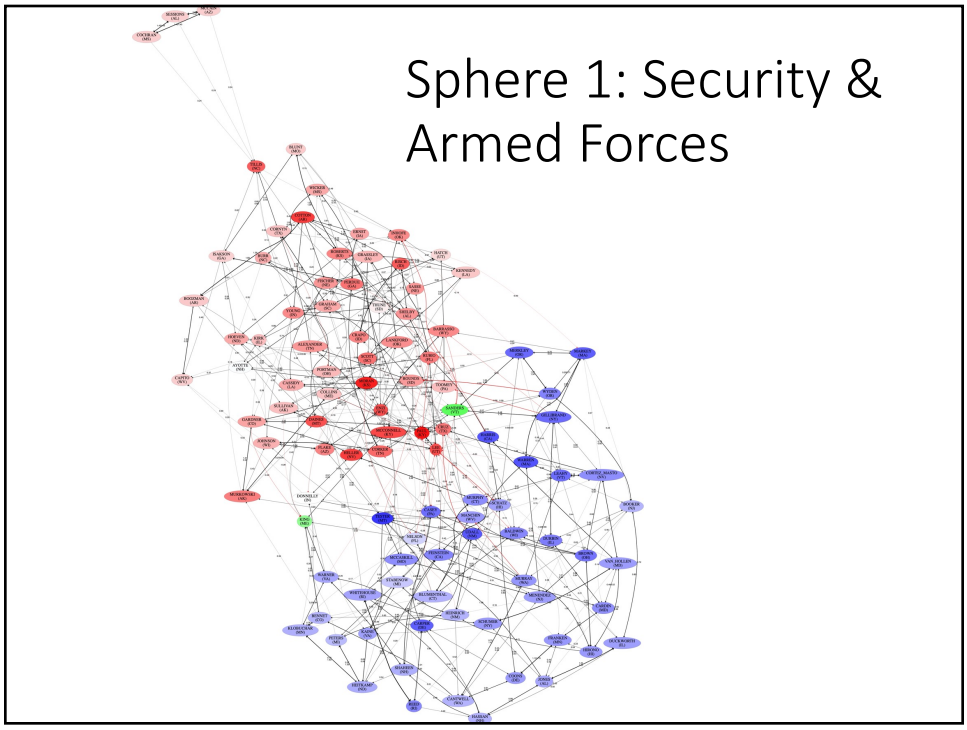

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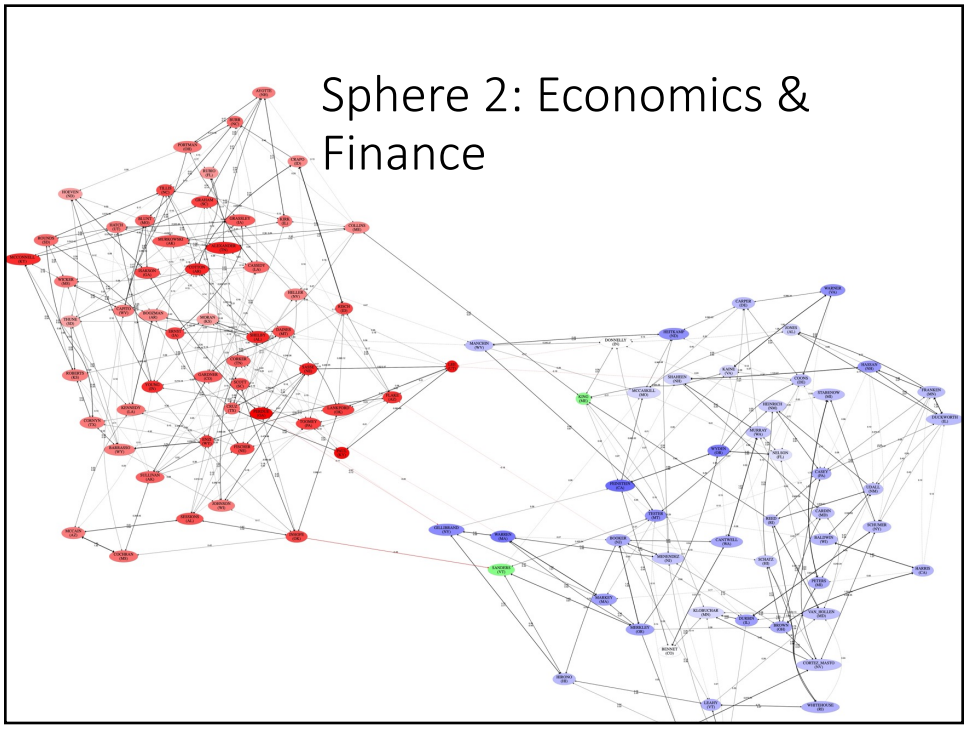


7

Bowdoin Effort Earns Top Award at International Computer Science Conference

Archives

July 31, 2018 by Tom Porter

Best Paper Award
AAMAS 2018

Professor Mohammad Irfan, in the middle, receives the Best Paper Award from AAMAS Program Chairs Gita Sukthankar (L) and Mehdi Dastani (R).

A research paper coauthored by a Bowdoin professor and one of his former students has earned the top spot at a recent computer science conference in Sweden. The paper employs computational game theory to model and predict congressional voting patterns. It was written by Assistant Professor of Digital and Computational Studies and Computer Science Mohammad Irfan and Tucker Gordon '17.

9

How Does Our Social Network Influence Our Behavioral Choices?

"No man is an island" wrote the poet John Donne in 1624, meaning whether we like it or not, we are all connected. It's an assertion that rings truer than ever in today's networked world, and a it's a central theme of the research currently being done by computer scientist Mohammad Irfan and his colleagues.

NSF Core Research Grant

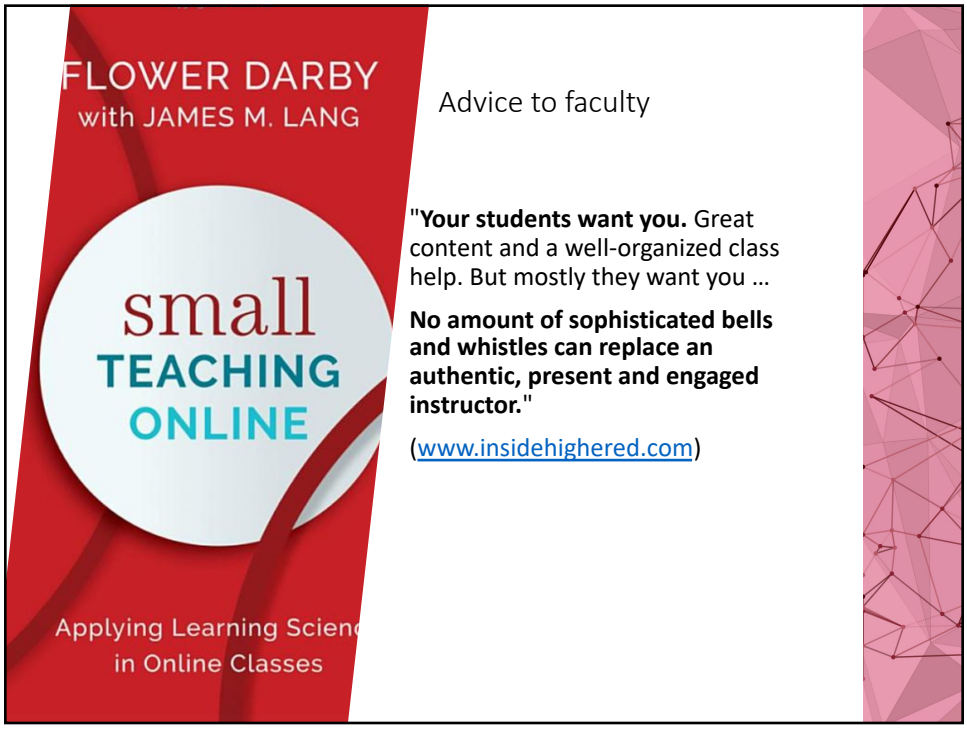
ssor of Digital and Computational
and Computer Science (CS) Irfan
d to secure around half a million
ng for an exciting multiyear
ng human interactions in
networks. The research could have implications for many fields, he says, from public health to energy pricing to finance to the analysis of congressional voting patterns.

The award was made by the National Science Foundation (NSF) and done in collaboration with Luis E. Ortiz of the University of Michigan—Dearborn, for a multiyear research initiative. It's all part of a core NSF program called Information and Intelligent Systems, says Irfan, who is the project director (while Bowdoin is the lead organization.)

10

He was very satisfied with his teaching career.
He said something that reminded me of my own students.
He said, “The students of Collegiate School wanted to learn beyond textbooks. And if the students are not interested in learning, then there’s no joy in teaching.”
What a beautiful thing to say! That’s why I’m itching to get into a class (even during my sabbatical).

12



Advice to faculty

"**Your students want you.** Great content and a well-organized class help. But mostly they want you …

No amount of sophisticated bells and whistles can replace an authentic, present and engaged instructor."

(www.insidehighered.com)

14

I teach humans

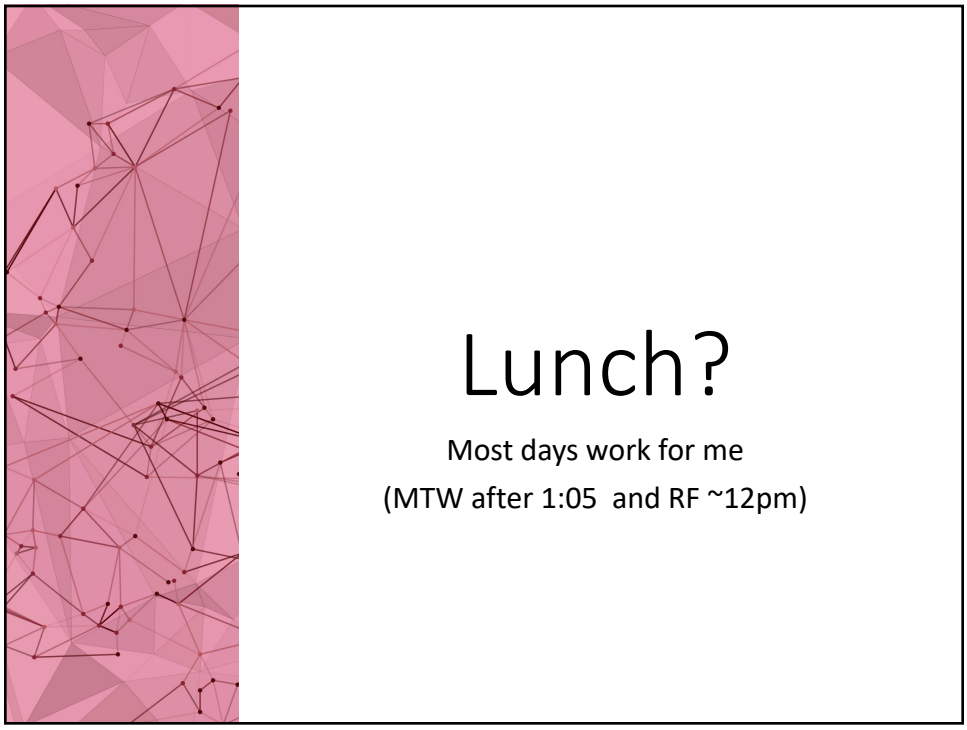
intellectually challenging courses with

care, compassion, and emotional engagement.

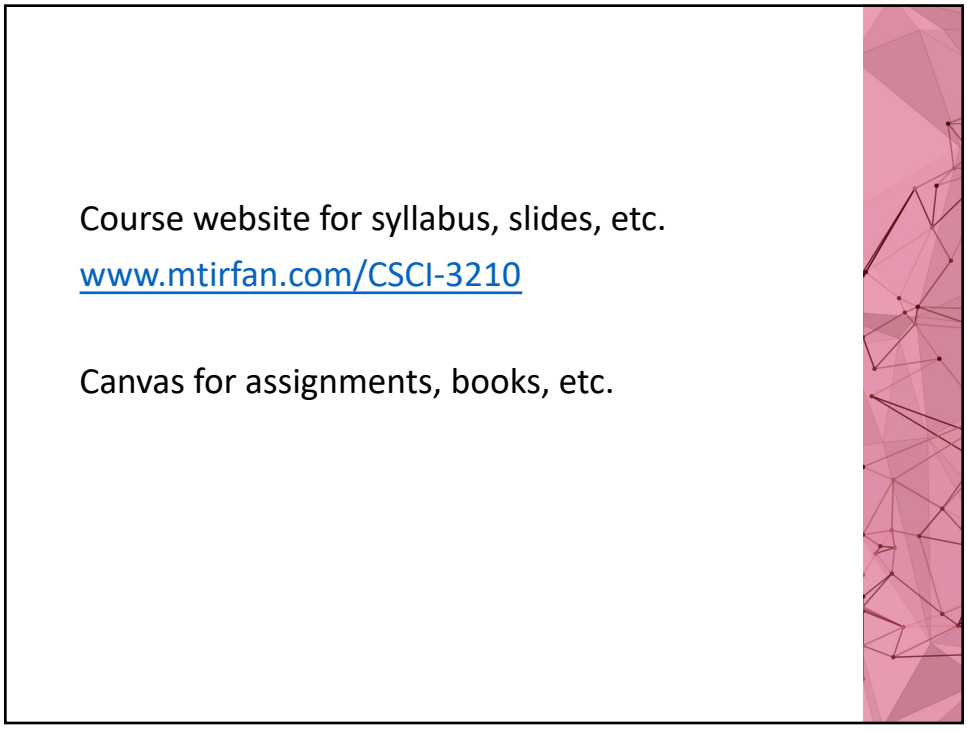
15

You

16


Lunch?
Most days work for me
(MTW after 1:05 and RF ~12pm)

17

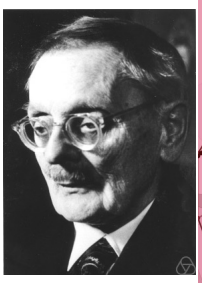

Course website for syllabus, slides, etc.
www.mtirfan.com/CSCI-3210
Canvas for assignments, books, etc.

18

Game Theory

- "Game"

  Ernst Zermelo (1913):
  In any chess game that does not end in a draw, a player has a winning strategy



- Mathematical theory of strategic decision making
  John von Neumann (1944)

20

Applications

- Application: market equilibria
  - Predict where the market is heading to
- Mechanism design and auctions
  - Google and Yahoo apply game-theoretic techniques
    - Keyword search auction
  - Spectrum allocation among wireless companies

21

21

Applications

Understanding the Internet: “Selfish routing” is a constant-factor off from optimal

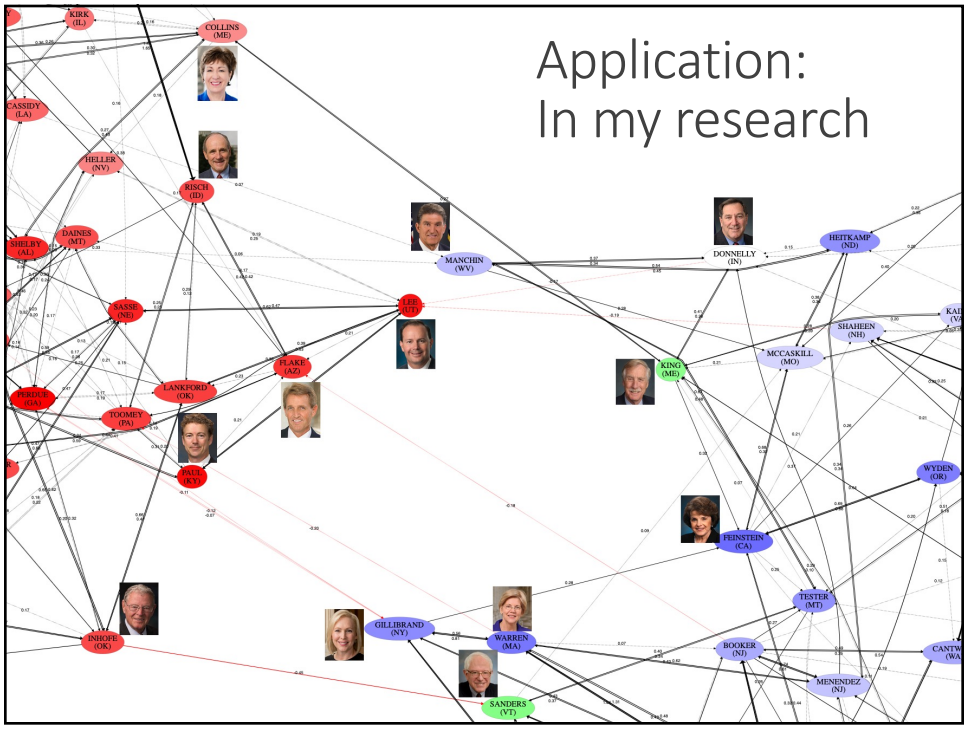
22

22

Applications

- Load balancing and resource allocation
- p2p and file sharing systems
- Cryptography and security
- AI: explainable AI
- Social and economic networks, etc.

23



24

The Power Of Context in Networks:
Ideal Point Models with Social Interactions

AAMAS 2018, Sweden

With Tucker Gordon'17

25

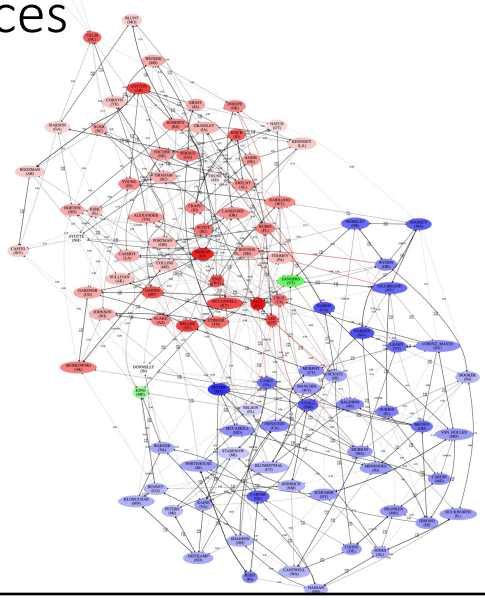
Spheres Of Legislation: Polarization And Most Influential Nodes In Behavioral Context

Complex Networks 2019, Portugal.

With Andrew Phillips'19 &
Luca Ostertag-hill'20

26

Sphere 1: Security & Armed Forces



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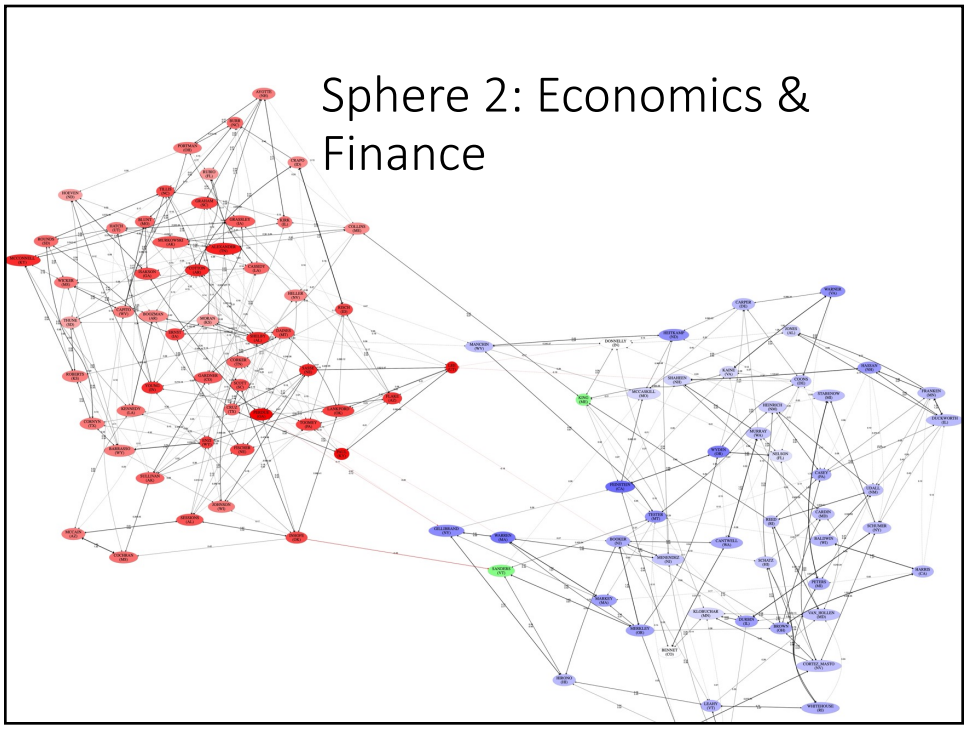
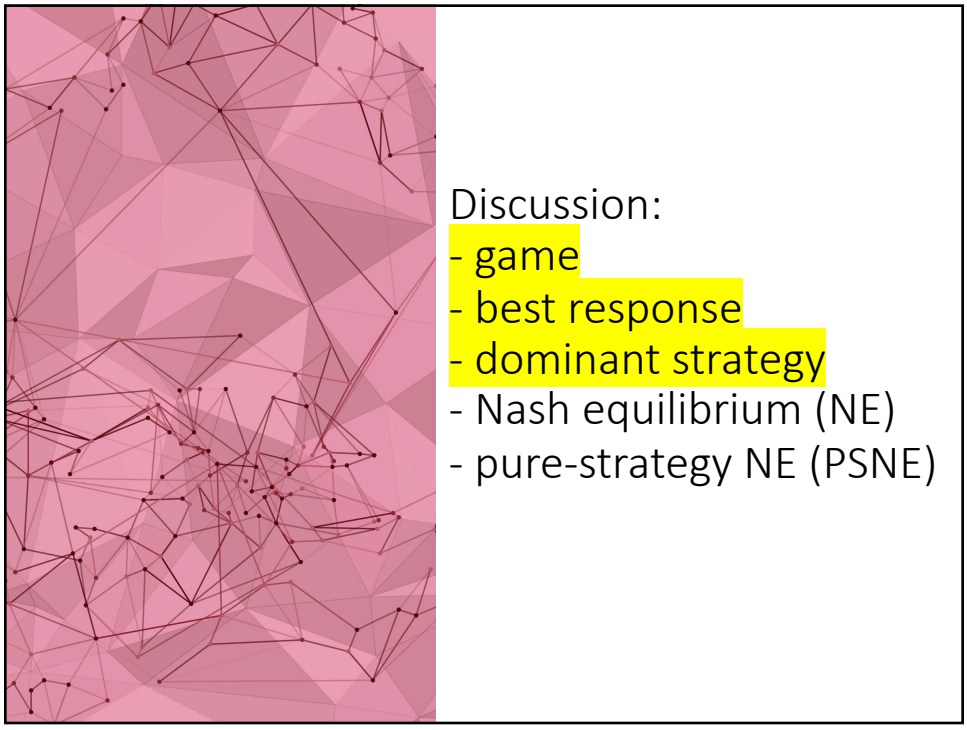

Sphere 2: Economics & Finance

28


Discussion:
- game
- best response
- dominant strategy
- Nash equilibrium (NE)
- pure-strategy NE (PSNE)

29

Example: Split or Steal

https://www.youtube.com/watch?v=yM38mRHY150

- Rules of the game
- Outcome

30

Game model of split or steal

- One-shot game (simultaneous move)
- 3 components
  - Players
  - Strategies/actions
  - Payoffs

31

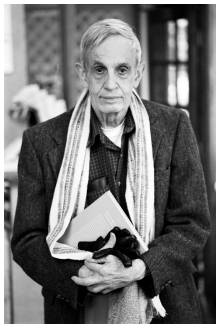
		Lucy	
	Payoff matrix	Split	Steal
Tony	Split	$33K, $33K	Frust., $66K
	Steal	$66K, Frust.	$0, $0

32

Why did they end up with 0?

		Lucy	
	Payoff matrix	Split	Steal
Tony	Split	$33K, $33K	Frust., $66K
	Steal	$66K, Frust.	$0, $0

Nash Equilibrium
Everyone plays their best response to others simultaneously



John F. Nash
Nobel Prize, 1994

33

Best response

- Best strategy of a player, given the other players' strategies
- Always exists!

34

(Strictly/weakly) dominant strategy

- A strategy of a player that is (strictly/weakly) better than any of their other strategies, no matter what the other players do
- Does not always exist

35

Famous example: prisoner's dilemma

		Suspect 2	
	Payoff matrix	Not Confess	Confess
Suspect 1	Not Confess	-1, -1	-10, 0
	Confess	0, -10	-4, -4

36

Drug usage in cycling

		Cyclist 2	
	Payoff matrix	No Drugs	Drugs
Cyclist 1	No Drugs	3, 3	1, 4
	Drugs	4, 1	2, 2

37

Example

(Weakly) dominant strategy

Neither is a dominant strategy

Column Player

Payoff matrix	L	R
U	10, 50	5, 0
D	0, 0	5, 10

Row Player

38

Checkpoint

- What is the difference between a dominant strategy and a best response?
- What is the difference between weakly and strictly dominant strategies? Will a player always have one?

39

Discussion:
- game
- best response
- dominant strategy
- Nash equilibrium (NE)
- pure-strategy NE (PSNE)

40

Nash equilibrium (NE)

- A joint strategy aka a strategy profile (one strategy/player) where every player plays their best response to others simultaneously
- (Equiv.) A joint strategy s.t. no player gains by deviating unilaterally
  - Useful for checking whether a joint strategy is NE

42

Pure-strategy Nash equilibrium (PSNE)

- Players do not use any probability in choosing strategies as they do in "mixed-strategy"
- Every player plays their “pure” best response to others simultaneously

43

Checkpoint

- What is the difference between best response and PSNE?
- Is there a connection between dominant strategy and PSNE?

44

Quiz

- Watch the following clip from the movie *a Beautiful Mind* portraying Nash's discovery of NE

https://www.youtube.com/watch?v=-6eK0yiw9t0

- Is this actually a Nash equilibrium?
  - Detailed answer: A blog post (also posted on the class website)

47

Misconceptions

- Equilibrium signifies a tie/draw/balance
- Equilibrium outcome is the best possible outcome for all players (*A Beautiful Mind*)
- Self-interested players want to hurt each other

48

Questions

- Does NE always exist? (Answer later ...)
- If it exists, is it unique?

49

Games with multiple NE

1. Battle of the sexes (Coordination)
2. Hawk-dove game (anti-coordination)

50

Does NE always exist?
Mixed-strategy NE (MSNE)

52

Penalty kick game

53

Penalty kick game (continued)

54

Penalty kick game (continued)

55

Penalty kick game: first model

Zero-sum Game

E[GK plays Left]	E[GK plays Right]
= p(1) + (1-p)(-1) = 2p – 1	= p(-1) + (1-p)(1) = 1 – 2p

2p – 1 = 1 – 2p
➔ p = ½

Similarly,
q = ½

Shooter \ Goalkeeper	Left (q)	Right (1-q)
Left (p)	-1, +1	+1, -1
Right (1-p)	+1, -1	-1, +1

p = ½ and q = ½ is an MSNE.
WHY?

56

Penalty kick game (real-world)

- *"Professionals Play Minimax"*- Ignacio Palacios-Huerta

Equilibrium probabilities (computed by solving equations) match real-world probabilities from data!

Shooter \ Goalkeeper	Left (0.42)	Right (0.58)
Left (0.38)	0.58, 0.42	0.95, 0.05
Right (0.62)	0.93, 0.07	0.70, 0.30

From real-world data

https://bleacherreport.com/articles/755195-champions-league-08-analyzing-one-of-the-most-iconic-shootouts-in-history

57

Definition: MSNE

A joint mixed strategy (given by each player's probability distribution over their actions) is an MSNE if

> no player can improve their expected payoff by unilateral deviation (that is, by changing their probability distribution).

58

What does mixed strategy mean?

- Active randomization – tennis, soccer
- Proportion interpretation – evolutionary biology
- Probabilities of player 1 are the beliefs of player 2 about what player 1 is doing (Bob Aumann)
- Misconception
  - ~~Players just choose probabilities~~
- Correct
  - players play pure strategies chosen according to these probabilities

59

Von Neumann's Theorem (1928)

Every finite 2-person zero-sum game has a mixed equilibrium

John von Neumann (1903 – 1957)

60

60

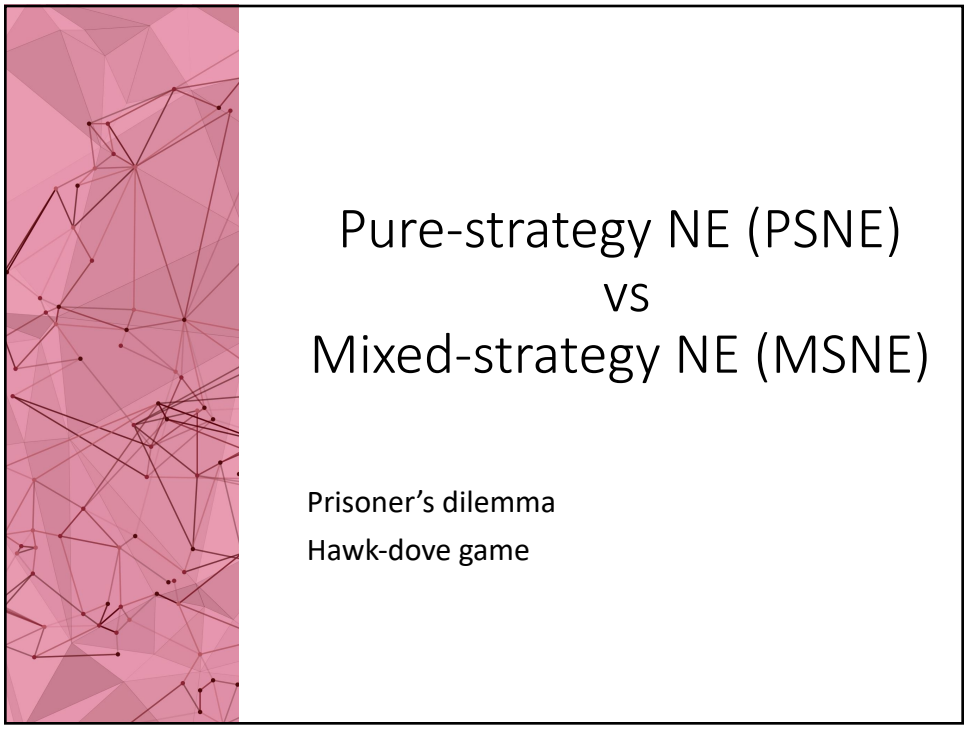
Theorem of Nash (1950)

- Every finite game has an equilibrium in mixed strategies
- Reading

John F. Nash (1928 – 2015)
Nobel Prize, 1994

61

61

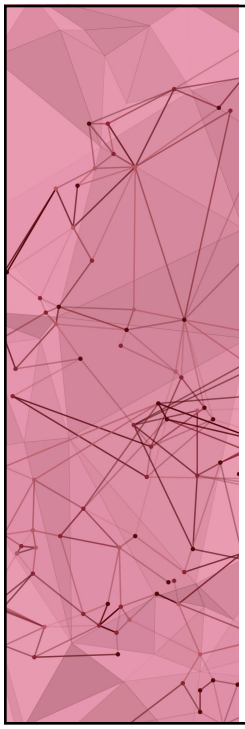


62

Key take-away messages

- Players act simultaneously, but NE outcome is stable in the sense that there is no incentive for unilateral deviation.
- There is always at least one MSNE (including PSNE). A PSNE is not guaranteed.
- The concept of NE doesn't say **how** NE happens.
- NE is not a balance or tie. It is often times a socially-inefficient outcome.

68



Formal/Mathematical Definitions

Book: Essentials of Game Theory

Ch 1, 2

70

Background: discrete math

- Set theory
  - Sets
  - Representation: listing members or properties
  - Belongs to
  - Cardinality
  - Subset
  - Empty set
  - Power set
  - Operations: union, intersection, difference, **product**
- Discrete Math: sum, product

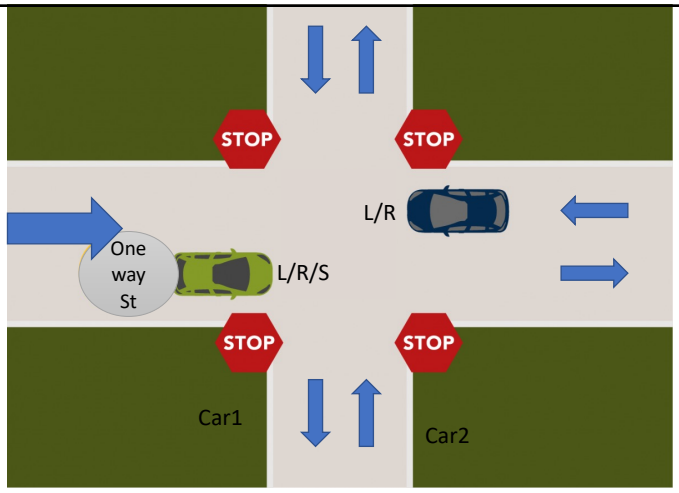
71

Normal form games

Definition 1.2.1 (Normal-form game). *A (finite,* n*-person)* normal-form game *is a tuple* (N, A, u)*, where:*

- N *is a finite set of* n *players, indexed by* i*;*
- $A = A_1 \times \cdots \times A_n$*, where* A_i *is a finite set of* actions *available to player* i*. Each vector* $a = (a_1, \ldots, a_n) \in A$ *is called an* action profile*;*
- $u = (u_1, \ldots, u_n)$ *where* $u_i : A \mapsto \mathbb{R}$ *is a real-valued* utility *(or* payoff*)* function *for player* i*.*

72



Car 2

Car 1		L	R
	L	1, 1	0, 0
	R	0, 0	1, 1
	S	0, 0	1, 1

Idea: Connor Marrs

Photo modified from topdriver.com

73

Zero-sum/constant-sum game

Definition 1.3.2 (Constant-sum game). *A two-player normal-form game is* constant-sum *if there exists a constant c such that for each ~~strategy profile~~ action profile $a \in A_1 \times A_2$ it is the case that $u_1(a) + u_2(a) = c$.*

Is constant sum the same as zero sum?

76

Example (zero-sum game)

- You and opponent flip two coins
- Same parity (both heads or both tails)=> you win
- Otherwise, opponent wins

		Opponent: Heads	Opponent: Tails
You	Heads	$1, -1$	$-1, 1$
	Tails	$-1, 1$	$1, -1$

77

Mixed-strategy NE

- Mixed strategy involves a player
- Mixed-strategy profile involves all players
- Expected utility
- Best response
- Nash equilibrium

78

Mixed strategy

Definition 1.4.1 (Mixed strategy). *Let (N, A, u) be a normal-form game, and for any set X let $\Pi(X)$ be the set of all probability distributions over X. Then the set of* mixed strategies *for player i is $S_i = \Pi(A_i)$.*

- A mixed strategy is an element of S_i

Definition 1.4.2 (Mixed-strategy profile). *The set of* mixed-strategy profiles *is simply the Cartesian product of the individual mixed-strategy sets, $S_1 \times \cdots \times S_n$.*

79

Expected utility

Definition 1.4.4 (Expected utility of a mixed strategy). *Given a normal-form game* (N, A, u)*, the expected utility* u_i *for player* i *of the mixed-strategy profile* $s = (s_1, \ldots, s_n)$ *is defined as*

$$u_i(s) = \sum_{a \in A} u_i(a) \prod_{j=1}^{n} s_j(a_j).$$

80

Best response

Definition 2.2.1 (Best response). *Player* i*'s* best response *to the strategy profile* s_{-i} *is a mixed strategy* $s_i^* \in S_i$ *such that* $u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$ *for all strategies* $s_i \in S_i$.

81

Nash equilibrium

Definition 2.2.2 (Nash equilibrium). *A strategy profile* $s = (s_1, \ldots, s_n)$ *is a* Nash equilibrium *if, for all agents* i, s_i *is a best response to* s_{-i}.

82

Reading [on class website]

- Nash's paper (1950): Every finite game has an equilibrium in mixed strategies
- Prof. Irfan's explanation

83

Other solution concepts

- Pareto optimal solution (Ch 6 of EK)
- Socially optimal solution (Ch 6 of EK)
- Correlated equilibria (Ch 1 of AGT)

86

Pareto opt and socially opt

		Player 2	
		L	R
Player 1	U	30, 0	5, 10
	D	20, 20	0, 30

87

Correlated equilibrium

Roger Myerson (Nobel laureate):

"If there is intelligent life on other planets, in a majority of them, they would have discovered correlated equilibrium before Nash equilibrium."

88

Intuition

Car 2

Car 1		Stop	Go
	Stop	0, 0	0, 1
	Go	1, 0	-100, -100

- What are the pure and mixed NE?
- What are their limitations?

89

Definition: correlated equilibrium (CE)

- A probability distribution p over *action profiles* such that whenever an action profile a is drawn according to p and each player i is individually told to play a_i:
  - Playing a_i is i's best response conditioned on seeing a_i.
  - That is, for any other action a_i' of i:

$$\sum_{a_{-i}} p(a_i, a_{-i})\, u_i(a_i, a_{-i}) \geq \sum_{a_{-i}} p(a_i, a_{-i})\, u_i(a_i', a_{-i}).$$

- Connection with NE?

Definition 1.4.4 (Expected utility of a mixed strategy). *Given a normal-form game (N, A, u), the expected utility u_i for player i of the mixed-strategy profile $s = (s_1, \ldots, s_n)$ is defined as*

$$u_i(s) = \sum_{a \in A} u_i(a) \prod_{j=1}^{n} s_j(a_j).$$

90

Computing NE and CE

93